

## Tutorial 7: Gravitational field

Given: Gravitational constant,  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

### A. Self-practice

- 1 A satellite is brought from the surface of the Earth to an orbit at a height of 144 km above Earth's surface. Taking the radius of the Earth to be 6371 km, show that its weight decreased by around 5%.

Soln: On Earth's surface,  $F_G = \frac{GMm}{r_E^2}$

On orbit in space,  $F_G' = \frac{GMm}{(r')^2}$

$$\frac{F_G'}{F_G} = \frac{r^2}{(r')^2} = \frac{(6371)^2}{(6371 + 144)^2} = 0.956$$

Hence  $F_G'$  at orbit is around 5% less than  $F_G$  on Earth's surface.

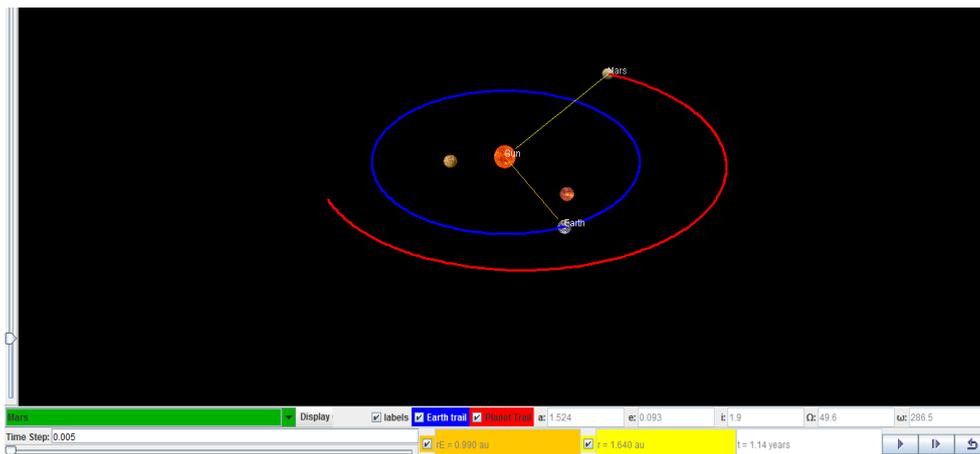
- 2 This ICT question is to help you understand and apply Kepler's 3rd law through the Solar System.

#### Apparatus required:

- Computer installed with Java runtime and the EJS java applet, titled "**Kepler System Model**", which can be downloaded from LMS or url:

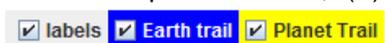
[https://dl.dropbox.com/u/44365627/lookangEJSworkspace/export/ejs\\_KeplerSystem3rdLaw03.jar](https://dl.dropbox.com/u/44365627/lookangEJSworkspace/export/ejs_KeplerSystem3rdLaw03.jar)

Through much trial and error scrutinising the planetary data, Kepler was able to find a connection between the period  $T$  of orbits of the planets in the Solar System and its mean radii  $r$  of orbits about the Sun which he set out in his Third Law about planetary system. Now put yourself in the shoes of Kepler and try to deduce the connection yourself using the following EJS.



Carry out the following steps to tabulate period  $T$  and radius  $r$  of the orbit, for the 5 planets, namely Mercury, Venus, Mars, Jupiter and Saturn:

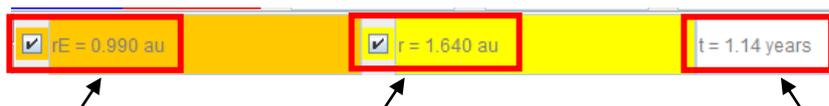
- Open the EJS java applet titled "**Kepler System Model**". This model allows you to observe the orbit of different planets (one planet at a time), around the Sun.
- Check the boxes for (i) labels – to display the descriptive texts for planets; (ii) Earth trail – to display the orbital path of Earth; (iii) Planet trail – to display the orbital path of planet, as shown:



- c) Select the planet you wish to observe using the 'Select Planet' drop-down menu, as shown below, starting with planet Mercury (the planet closest to Sun, with the shortest radius of orbit):



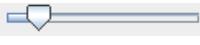
- d) Orientate your point of view by 'left-click & hold' on the model and move your mouse to view the Solar System in different 3D perspectives.
- e) The radius of orbit for Earth  $r_E$  is about 1 au. (in astronomical unit,  $1 \text{ au} = 1.496 \times 10^{11} \text{ m}$ ), The radius  $r$  of orbit (in au) and duration of orbit (in years) for the planet are displayed as shown:



radius of orbit for Earth,  $r_E$  (in au)

radius of orbit for planet,  $r$  (in au)

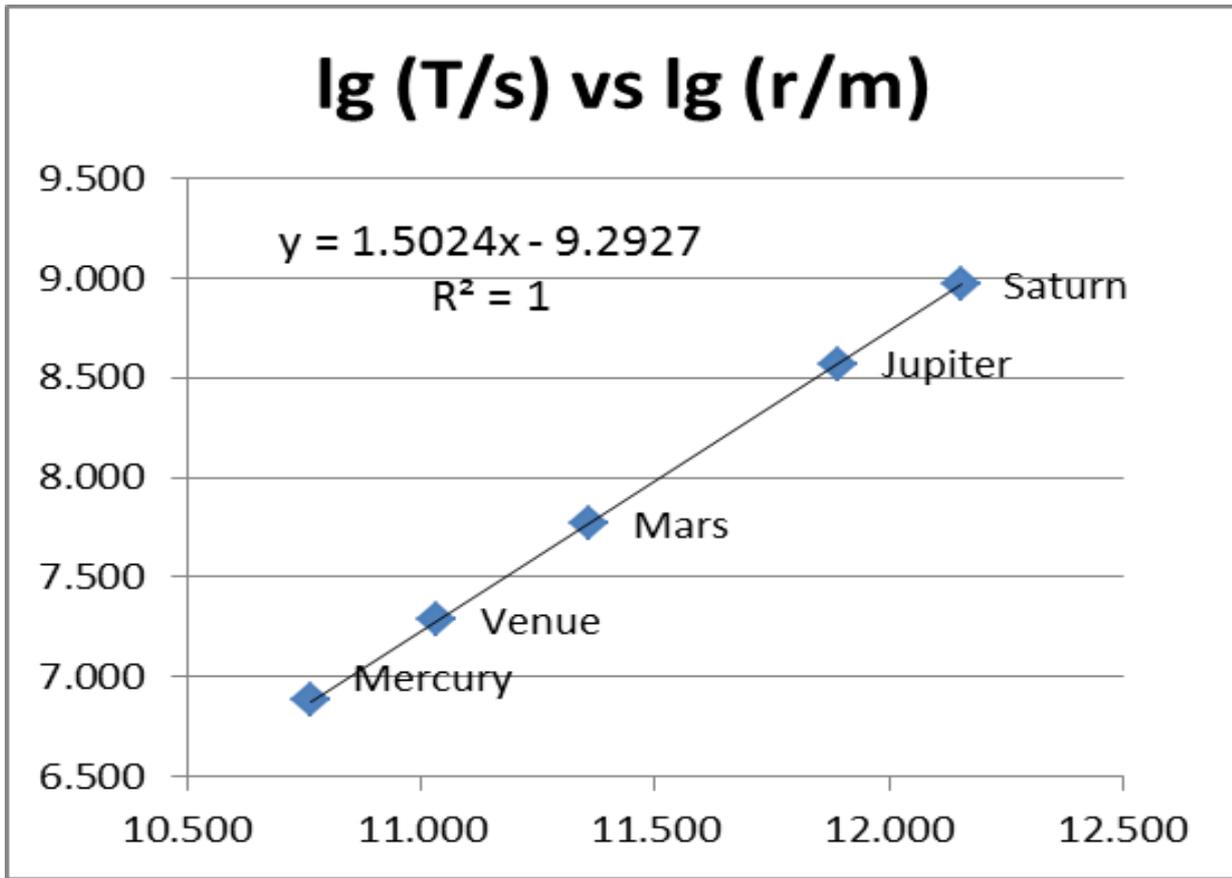
Duration of orbit (in years)

- f) Check the boxes for  $r_E$  and  $r$  to display the radii of orbits. The time step can be adjusted by using the slider **Time Step:**  to control the speed of simulation (i.e. how fast the simulation runs, not the angular speed of planet!). To watch the simulation slowly at step by step run, click the  button one click at a time. Do you notice that the planetary motion is not exactly circular?
- g) Record the average radius  $r$  of orbit for planet Mercury and **convert the value from 'au' to 'm'**.
- h) To determine the period of orbit  $T$  for planet Mercury, ensure that the model is reset by clicking the  button such that the duration of orbit starts from 0.00 year. Play the simulation by clicking  button and click  button when planet Mercury has rotated exactly one round. (*Hint: use the  button when the planet almost completes one round*) Record the duration of orbit (for one round) as the period  $T$  of orbit (in years) for planet Mercury. **Convert  $T$  in years to seconds.**
- i) Repeat step 3 to 8 for the other four planets, namely Venus, Mars, Jupiter and Saturn.
- j) Tabulate the results below.

Planet	$T/\text{year}$	$T/\text{s}$	$r/\text{au}$	$r/\text{m}$	$\lg(r/\text{m})$	$\lg(T/\text{s})$
Mercury	0.24	7.57E+06	0.388	5.80E+10	10.764	6.879
Venus	0.61	1.92E+07	0.723	1.08E+11	11.034	7.284
Mars	1.87	5.90E+07	1.524	2.28E+11	11.358	7.771
Jupiter	11.8	3.72E+08	5.209	7.79E+11	11.892	8.571
Saturn	29.5	9.30E+08	9.527	1.43E+12	12.154	8.969

- k) According to Kepler, for planets performing orbits about the sun, the period  $T$  of orbit is related to the radius  $r$  of the orbit by the equation  $T = Ar^n$ , where  $A$  and  $n$  are constants.
- (i) Plot a suitable graph to deduce the values of ' $n$ '. (Hint: refer to the headings given in the table)
- (ii) Kepler's Third law states that "the square of the orbital periods ( $T$ ) is *directly proportional* to the cube of their orbital radii ( $r$ )."

Check whether your value of  $n$  found in (a) is consistent with Kepler's Third Law.



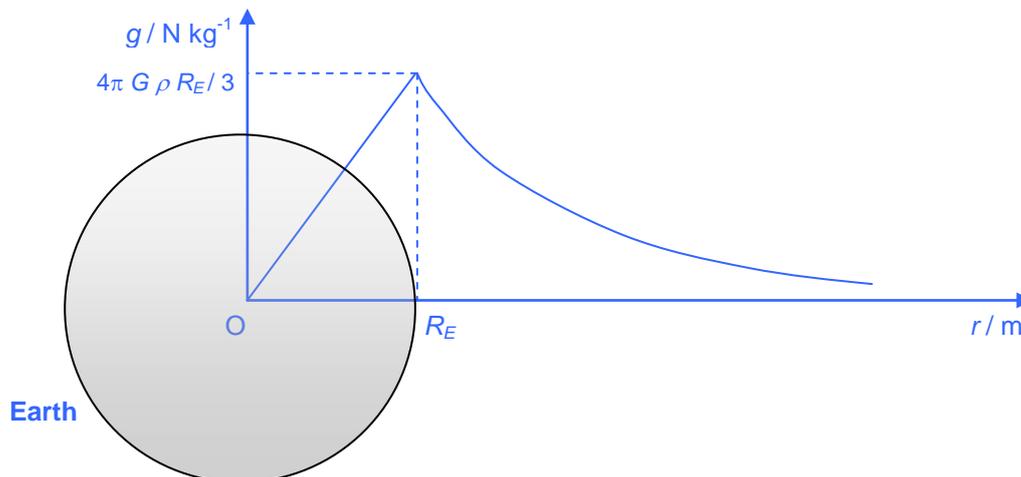
Since  $n = 1.5$  (ie  $3/2$ ), Kepler's Third law is verified.

**Discussion**

**Gravitational force and field strength**

- 3 Sketch a fully labelled graph showing the relationship between the Earth's gravitational field strength,  $g$ , and the distance away from its centre of gravity,  $r$ .

Soln: Let  $O$  be the centre of the Earth and  $R_E$  be its radius.  
 Beneath the Earth's surface,  $g = GM/r^2$  where  $M = \rho(4\pi r^3/3)$ .  
 Hence  $g = 4\pi G\rho r/3$ , i.e.  $g \propto r$  when  $r \leq R_E$   
 Above the Earth's surface,  $g$  can be treated as if the entire mass of the Earth is concentrated at its centre. Mass of Earth is a constant, and hence  $g \propto 1/r^2$  for  $r \geq R_E$



- 4 At a point on the surface of a uniform sphere of diameter  $d$ , the gravitational field due to the sphere is  $X$ . Calculate the corresponding gravitational field value on the surface of a uniform sphere of the same density but of diameter  $2d$ .

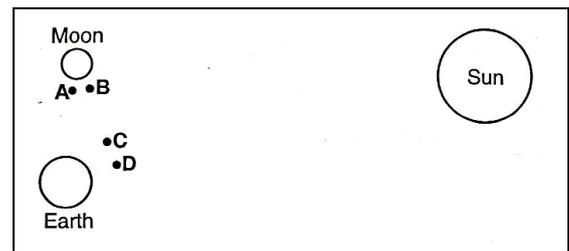
Soln: Consider  $g = \frac{GM}{r^2}$

$$\therefore g = \frac{G\left(\frac{4\pi r^3}{3}\right)(\rho)}{r^2} = \frac{4\pi G r \rho}{3} = \frac{2\pi G d \rho}{3}$$

$$\therefore g \propto d$$

Hence, if diameter increases twice to  $2d$ ,  $g$  will increase twice to  $2X$ .

- 5 [N10/I/15]  
The neutral point in the gravitational field between the Sun, the Earth and the Moon is the point at which the resultant gravitational field due to the three bodies is zero. The mass of the Earth is about 80 times the mass of the Moon. At what position is it possible for the neutral point to be? (The diagram is not drawn to scale.)



**Answer : B**

If we consider the Earth-Moon system only, the neutral point would be at point A.

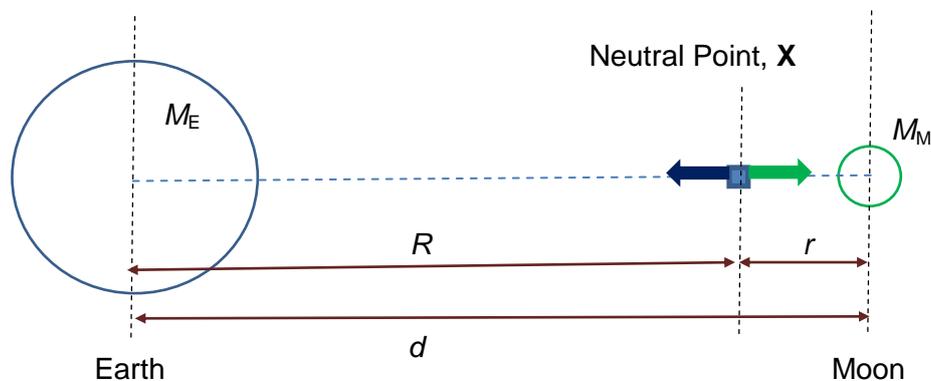
However, when we consider the gravitational attractive force of the Sun, the neutral point is shifted slightly to B.

Learning point:

*The vector sum of the gravitational field strengths at the neutral point is zero.*

*The neutral point always lies closer to the smaller mass.*

- 6 The mass of the Earth is  $6.0 \times 10^{24}$  kg and the mass of the Moon is  $7.4 \times 10^{22}$  kg. The distance between their centres is  $3.8 \times 10^8$  m and the radius of the Earth is 6371 km while that of the Moon is 1737 km.  
The position of the gravitational neutral point X, lies between the Earth and the Moon.  
(a) Show that the gravitational neutral point X divides the distance between the Earth and the Moon roughly in the ratio 9:1 being closer to the Moon.  
(b) Calculate the distance between the neutral point X and the centre of the Moon.  
[ $3.8 \times 10^7$  m]



Let  $R$  = distance between the neutral point and centre of Earth  
Let  $r$  = distance between the neutral point and centre of Moon.

Soln: (a)  $\frac{GM_E}{R^2} = \frac{GM_M}{r^2}$

$$\frac{R^2}{r^2} = \frac{M_M}{M_E}$$

$$\frac{R}{r} = \sqrt{\frac{M_M}{M_E}} = \sqrt{\frac{6.0 \times 10^{24}}{7.4 \times 10^{22}}} = 9.00$$

(b)  $R = 9.0r$  since  $R = d - r$

$$\frac{d - r}{r} = 9.0$$

$$\frac{d}{r} - 1 = 9.0$$

$$\frac{d}{r} = 10.0$$

$$r = \frac{d}{10.0} = \frac{3.8 \times 10^8}{10.0} = 3.8 \times 10^7 \text{ m}$$

### Learning point

The net force is zero at the neutral point X as the gravitational force due to the Earth is equal and opposite that due to the Moon. This is the consequence of Newton's 1st Law. The neutral point is always closer to the smaller mass.

### Gravitational potential energy & potential

- 7 (a) Calculate the minimum speed required to project a body from Earth to somewhere very far from Earth (and never return to Earth). Give your answer in 5 s.f. Take the radius of the Earth to be 6371 km and mass of the Earth to be  $5.97 \times 10^{24}$  kg.
- (b) Does the escape speed depend on the mass of the body?
- (c) What is the speed of a body, released from a very large distance from Earth, as it reaches Earth surface? [1.1181 x 10<sup>4</sup> m s<sup>-1</sup>]

Soln:

(a) To determine the escape speed,  $v$ , from Earth:

Assuming negligible atmospheric friction,

Minimum KE from projection  $\geq$  Change in GPE for body to reach infinity

$$\text{i.e. } \frac{1}{2}mv^2 \geq [0 - (-GM_E m/R_E)]$$

$$\therefore \frac{1}{2}mv^2 \geq GM_E m/R_E$$

$$v^2 \geq 2GM_E/R_E$$

$$v \geq \sqrt{2GM_E/R_E}$$

$$v \geq \sqrt{\frac{2(6.67 \times 10^{-11})(5.97 \times 10^{24})}{6.371 \times 10^6}}$$

$$v \geq 1.1181 \times 10^4 \text{ ms}^{-1}$$

Minimum projection speed = 1.1181 x 10<sup>4</sup> m s<sup>-1</sup>

(b) No, the escape speed does not depend on the mass of the body.

(c) The speed of the body will be  $1.1181 \times 10^4 \text{ m s}^{-1}$  when it reaches Earth surface.

- 8 A stationary object is released from a point at a distance  $3 R_M$  from the centre of the Moon which has radius  $R_M$  and mass  $M_M$ . Express the speed of the object just before hitting the Moon, in terms of  $G$ ,  $M_M$  and  $R_M$ .

Soln: Let mass of object be  $m$  and its speed just before hitting the Moon be  $v$ .

Loss in GPE = Gain in KE

$$\left(-\frac{GM_M m}{3R_M}\right) - \left(-\frac{GM_M m}{R_M}\right) = \frac{1}{2} m v^2 - 0$$

$$\left(\frac{2GM_M m}{3R_M}\right) = \frac{1}{2} m v^2$$

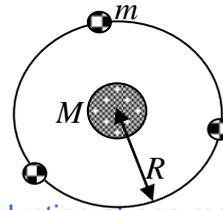
$$\therefore v^2 = \frac{4GM_M}{3R_M}$$

$$\text{i.e. } v = \left(\frac{4GM_M}{3R_M}\right)^{1/2}$$

- 9 An unknown planet of mass  $M$  has three identical moons of mass  $m$ , each moving with the same orbital speed in the same circular orbit of radius  $R$ . The moons are equally spaced so as to form an equilateral triangle configuration.

Show that the total gravitational potential energy,  $E_P$ , of this planet-moons system, is given

$$\text{by } E_P = -\frac{3Gm}{R} \left(M + \frac{m}{\sqrt{3}}\right).$$



Soln: With planet in place, GPE attributed to introduction of one moon,  $U_1 = -\frac{GMm}{R}$

GPE attributed to introduction of second moon,

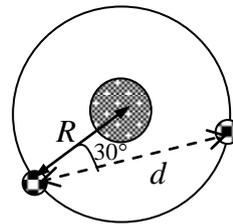
$$U_2 = \left(-\frac{GMm}{R}\right) + \left(-\frac{Gmm}{d}\right)$$

$$= \left(-\frac{GMm}{R}\right) + \left(-\frac{Gm^2}{\sqrt{3}R}\right)$$

GPE attributed to introduction of third moon,

$$U_3 = \left(-\frac{GMm}{R}\right) + 2\left(-\frac{Gmm}{d}\right)$$

$$= \left(-\frac{GMm}{R}\right) + \left(-\frac{2Gm^2}{\sqrt{3}R}\right)$$



$$d = 2R \cos 30^\circ = \sqrt{3}R$$

Hence, total GPE of system =  $U_1 + U_2 + U_3$

$$= \left(-\frac{3GMm}{R}\right) + \left(-\frac{3Gm^2}{\sqrt{3}R}\right)$$

$$= -\frac{3Gm}{R} \left(M + \frac{m}{\sqrt{3}}\right)$$

### Circular Orbits



- 10 A certain star of mass  $M$  and radius  $r$  rotates so rapidly that all objects at its equator just break contact with the planet's surface. Given that the gravitational constant is  $G$ , express the period of rotation  $T$  in terms of  $r$ ,  $M$  and  $G$ .

Soln: For object to just break contact  $\Rightarrow$  normal contact force on that object is zero  
 $\therefore$  Gravitational force on object = Centripetal force

$$\frac{GMm}{r^2} = mr\omega^2$$

$$\frac{GMm}{r^2} = mr\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = 4\pi^2 r^3 / (GM)$$

$$\text{i.e. } T = 2\pi \sqrt{(r^3 / MG)}$$

- 11 A satellite of mass 1800 kg is placed in an orbit, above the equator, at a distance  $4.22 \times 10^7$  m from the centre of the Earth. If the Earth's radius is  $6.37 \times 10^6$  m, and the acceleration of free fall at the surface of the Earth is  $9.81 \text{ m s}^{-2}$  (mass of Earth not given),

- calculate the angular velocity of the satellite,
- deduce whether it's possible that the satellite is in a geostationary orbit, and
- determine the speed, kinetic energy and hence the total energy of the satellite.
- Explain the significance of the sign in your answer for the total energy of the satellite.  
 $[7.28 \times 10^{-5} \text{ rad s}^{-1}; 3.07 \text{ km s}^{-1}; 8.49 \times 10^9 \text{ J}; -8.49 \times 10^9 \text{ J}]$

Soln: (a) Consider  $g = \frac{GM}{r^2}$  on earth's surface,

$$9.81 = GM / (6.37 \times 10^6)^2$$

$$GM = 3.981 \times 10^{14}$$

$$\text{Using } \frac{GMm}{r^2} = mr\omega^2$$

$$\omega = (GM/r^3)^{1/2} = [(3.981 \times 10^{14}) / (4.22 \times 10^7)^3]^{1/2}$$

$$= 7.278 \times 10^{-5} = 7.28 \times 10^{-5} \text{ rad s}^{-1}$$

- (b) Consider  $T = 2\pi / \omega$

$$T = 2\pi / (7.278 \times 10^{-5}) = 8.63 \times 10^4 \text{ s} = 24.0 \text{ hrs}$$

Orbital period of satellite = Rotational period of Earth

Hence it is possible that this satellite is placed in a geostationary orbit.

- (c) Speed of satellite,  $v = r\omega = (4.22 \times 10^7) (7.278 \times 10^{-5}) = 3071 \text{ m s}^{-1}$   
 $= 3.07 \text{ km s}^{-1}$

$$\text{Kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2}(1800)(3071)^2 = 8.49 \times 10^9 \text{ J}$$

$$\text{Recall that } \frac{GMm}{r^2} = \frac{mv^2}{r}, \text{ i.e. } \frac{1}{2} \frac{GMm}{r} = \frac{1}{2} mv^2$$

$$\therefore \text{Total energy, } E = U + E_K = -E_K = -8.49 \times 10^9 \text{ J}$$

- (d) The negative sign implies that the satellite does not have enough energy to escape the gravitational field of Earth, and is therefore bound to the Earth.

- 12 N99/III/2

- (b) A satellite P of mass 2400 kg is placed in a geostationary orbit at a distance of  $4.23 \times 10^7$  m from the centre of the Earth.

- Explain what is meant by the term *geostationary orbit*.
- Calculate
  - the angular velocity of the satellite,



2. the speed of the satellite,
  3. the acceleration of the satellite,
  4. the force of attraction between the Earth and the satellite,
  5. the mass of the Earth.
- (c) Explain why a geostationary satellite
- (i) must be placed vertically above the equator,
  - (ii) must move from west to east.
- (d) Why is a satellite in a geostationary orbit often used for telecommunication?  
[ $7.27 \times 10^{-5} \text{ rad s}^{-1}$ ;  $3.08 \text{ km s}^{-1}$ ;  $0.224 \text{ m s}^{-2}$ ;  $537 \text{ N}$ ;  $6.00 \times 10^{24} \text{ kg}$ ]

Soln: (b)(i) Geostationary orbit is one orbit in which a satellite moves such that it is always positioned above a particular place on the Earth's surface.

(ii)1.  $\omega = 2\pi/T = 2\pi / (24 \times 60 \times 60) = 7.272 \times 10^{-5} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$

2.  $v = r\omega = (4.23 \times 10^7)(7.272 \times 10^{-5}) = 3.08 \text{ km s}^{-1}$

3.  $a = r\omega^2 = (4.23 \times 10^7)(7.272 \times 10^{-5})^2 = 0.224 \text{ m s}^{-2}$

4.  $F_G = F_C = ma = (2400)(0.224) = 536.9 = 537 \text{ N}$

5.  $F_G = GMm/r^2 = 536.9 \text{ N}$   
 $\therefore \text{Mass of Earth, } M = (536.9)(4.23 \times 10^7)^2 / (6.67 \times 10^{-11})(2400)$   
 $= 6.00 \times 10^{24} \text{ kg}$

- (c)(i) So that the satellite will revolve in a path parallel to the Earth's equator or revolve about the same axis of rotation as the Earth's.

From Examiner's report:

*(Draw a sketch)*

*Gravitational force exerted on the satellite that provides for the centripetal force is acting towards the centre of the Earth. Any circular orbit must have its centre at the centre of Earth.*

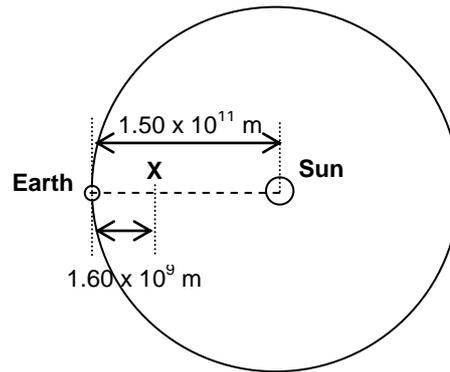
*Therefore, if the satellite were to be in orbit other than that in the equatorial plane (as sketched), it will sometimes be over the northern hemisphere and sometimes over the southern hemisphere. In such a case, the satellite cannot be geostationary.*

- (ii) The Earth rotates from West to East. For the geostationary satellite to appear at a fixed position above the Earth's surface, it must also move in the same direction as the Earth's rotation.

- (d) Geostationary satellite allows signals to be transmitted steadily since its position is fixed with respect to the transmitting stations and receiving stations on Earth.

13 N2000/III/2

- (a) (i) Define *angular velocity* for an object travelling in a circle.  
 (ii) Calculate the angular velocity of the Earth in its orbit around the Sun. Assume that the orbit is circular and give your answer in radians per second.
- (b) In order to observe the Sun continuously, a satellite of mass  $425 \text{ kg}$  is at point X, a distance of  $1.60 \times 10^9 \text{ m}$  from the centre of the Earth, as shown below.



Given: mass of Sun =  $1.99 \times 10^{30} \text{ kg}$   
 mass of Earth =  $5.98 \times 10^{24} \text{ kg}$   
 Earth-Sun distance =  $1.50 \times 10^{11} \text{ m}$

- (i) Calculate, using the data given,
  1. the pull of the Earth on the satellite,
  2. the pull of the Sun on the satellite.
- (ii) Sketch a similar figure as the above diagram, to show the relative positions of the Earth, the Sun and the satellite. On your sketch, draw arrows to represent the two forces acting on the satellite. Label the arrows with the magnitude of the forces.
- (iii) Calculate
  1. the magnitude and direction of the resultant force on the satellite,
  2. the acceleration of the satellite.
- (iv) The satellite is in a circular orbit around the Sun. Calculate the angular velocity of the satellite.
- (v) Using your answer to (a) (ii), describe the motion of the satellite relative to the Earth. Suggest why this orbit around the Sun is preferable to a satellite orbit around the Earth.
- (vi) Suggest two disadvantages of having a satellite in this orbit.  
 $[1.99 \times 10^{-7} \text{ rad s}^{-1}; 0.0662 \text{ N}; 2.56 \text{ N}; 2.49 \text{ N}; 5.87 \times 10^{-3} \text{ m s}^{-2}; 1.99 \times 10^{-7} \text{ rad s}^{-1}]$

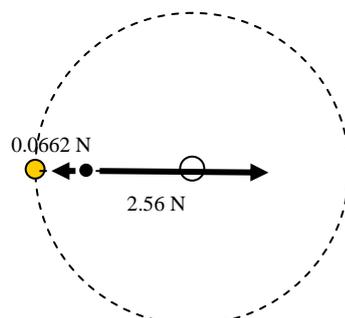
Soln: (a)(i) Angular velocity is defined as the rate of change of angular displacement of the object.

$$(ii) \quad \omega = 2\pi/T = 2\pi / (365 \times 24 \times 60 \times 60) = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

$$(b)(i) \quad 1. F_E = GM_E m / r^2 = (6.67 \times 10^{-11})(5.98 \times 10^{24})(425) / (1.60 \times 10^9)^2 = 0.0662 \text{ N}$$

$$2. F_S = GM_S m / r^2 = (6.67 \times 10^{-11})(1.99 \times 10^{30})(425) / (1.50 \times 10^{11} - 1.60 \times 10^9)^2 = 2.56 \text{ N}$$

(ii)



- (iii) 1. Resultant force =  $2.56 - 0.0662 = 2.494 = 2.49 \text{ N}$  (towards the Sun)
2. Acceleration =  $2.494/425 = 5.868 \times 10^{-3} = 5.87 \times 10^{-3} \text{ m s}^{-2}$
- (iv)  $a = r\omega^2$   
 $\therefore \omega = [(5.868 \times 10^{-3})/(1.484 \times 10^{11})]^{0.5} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$
- (v) Since the satellite and the Earth have the same angular velocity when moving about the Sun, the satellite will appear stationary with respect to the Sun.
- This orbit is preferred as it is located at a fixed distance from the centre of the Sun which allows the study of the Sun to be more reliable.
- (vi) The satellite is located closer to the Sun than Earth which means that it needs to be manufactured such as to withstand the stronger light intensity from the Sun. This could lead to high maintenance cost. Also, this satellite's orbit is much further from the Earth's surface than a typical geostationary orbit. Hence, the cost of launching the satellite could be higher.

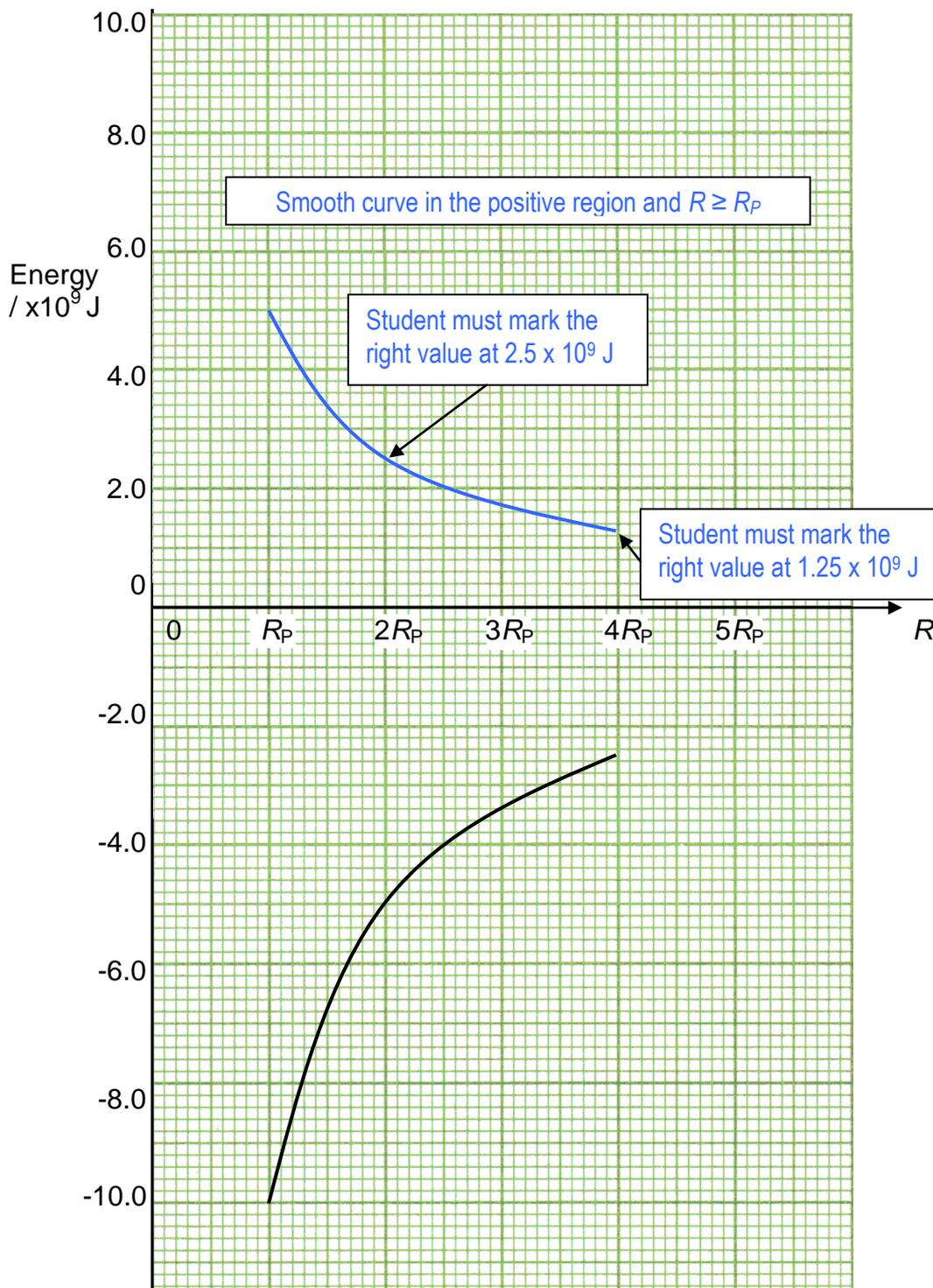
**14** 2008/II/3

A satellite of mass  $m$  orbits a planet of mass  $M$  and radius  $R_p$ . The radius of the orbit is  $R$ . The satellite and the planet may be considered to be point masses with their masses concentrated at their centres. They may be assumed to be isolated in space.

- (a) (i) Derive an expression, in terms of  $M$ ,  $m$  and  $R$ , for the kinetic energy of the satellite. Explain your working.
- (ii) Show that, for the satellite in orbit, the ratio  

$$\frac{\text{Gravitational Potential Energy of satellite}}{\text{Kinetic Energy of satellite}}$$
is equal to  $-2$ .

- (b) The variation with orbital radius  $R$  of the gravitational potential energy of the satellite is shown in the following graph.



- (i) On the same axes, draw the variation with orbital radius of the kinetic energy of the satellite. Your line should extend from  $R = 1.5 R_p$  to  $R = 4 R_p$ .
- (ii) The mass  $m$  of the satellite is 1600 kg. The radius of the orbit of the satellite is changed from  $R = 4 R_p$  to  $R = 2 R_p$ . Use the graphs to determine the change in orbital speed of the satellite. [520  $\text{m s}^{-1}$ ]

Soln:

$$(ai) \quad \Sigma F = \frac{mv^2}{r},$$

$$G \frac{Mm}{R^2} = \frac{mv^2}{R}$$

$$E_K = \frac{GMm}{2R}$$

$$(aii) \quad E_P = -G \frac{Mm}{r} \quad (\text{Negative sign is essential})$$

$$\frac{E_P}{E_K} = -2.0$$

(bi) Refer to graph

$$(bii) \quad \text{At } R = 4 R_P, E_k = 1.25 \times 10^9 \text{ J}, \\ \frac{1}{2} mv^2 = 1.25 \times 10^9 \text{ J} \\ v = 1250 \text{ m s}^{-1}$$

$$\text{At } R = 2 R_P, E_k = 2.5 \times 10^9 \text{ J}, \\ \frac{1}{2} mv^2 = 2.5 \times 10^9 \text{ J} \\ v = 1770 \text{ m s}^{-1}$$

$$\text{Change in speed} \quad = 1770 - 1250 \\ = 520 \text{ m s}^{-1}$$

Examiner's Comment

The last part proved to be a very good discriminator. The majority were able to determine the correct values for the kinetic energy at the two points. There were some who then forgot the  $10^9$  factor.

A significant majority then went on to calculate the change in speed using the expression  $\frac{1}{2} m \Delta v^2 = \Delta E_k$  giving a maximum mark of 3.

A significant minority of candidates scored zero marks as they equated the change in potential energy with the change in kinetic energy.

**B. Challenging (Optional)**

- 15 (a) With reference to the answers in **Q6 & Q7**, calculate the minimum required speed for a body to leave the surface of Earth and reach the surface of Moon. Give your answer in 5 s.f.  $[1.1076 \times 10^4 \text{ m s}^{-1}]$

Soln: To determine the required speed,  $v$ , from Earth to Moon:

Assuming negligible atmospheric friction,

Minimum KE from projection  $\geq$  Change in GPE for body to reach neutral point X

$$\text{i.e. } \frac{1}{2} mv^2 \geq [-GM_{EM}/R - GM_Mm/r] - [-GM_{EM}/R_E - GM_Mm/(d - R_E)]$$

$$\therefore \frac{1}{2} v^2 \geq [GM_{EM}/R_E + GM_Mm/(d - R_E)] - [GM_{EM}/R + GM_Mm/r]$$

$$v \geq \sqrt{2(6.67 \times 10^{-11}) \left[ \left( \frac{6 \times 10^{24}}{6.371 \times 10^6} + \frac{7.4 \times 10^{22}}{3.74 \times 10^8} \right) - \left( \frac{6 \times 10^{24}}{3.42 \times 10^8} + \frac{7.4 \times 10^{22}}{3.8 \times 10^7} \right) \right]}$$

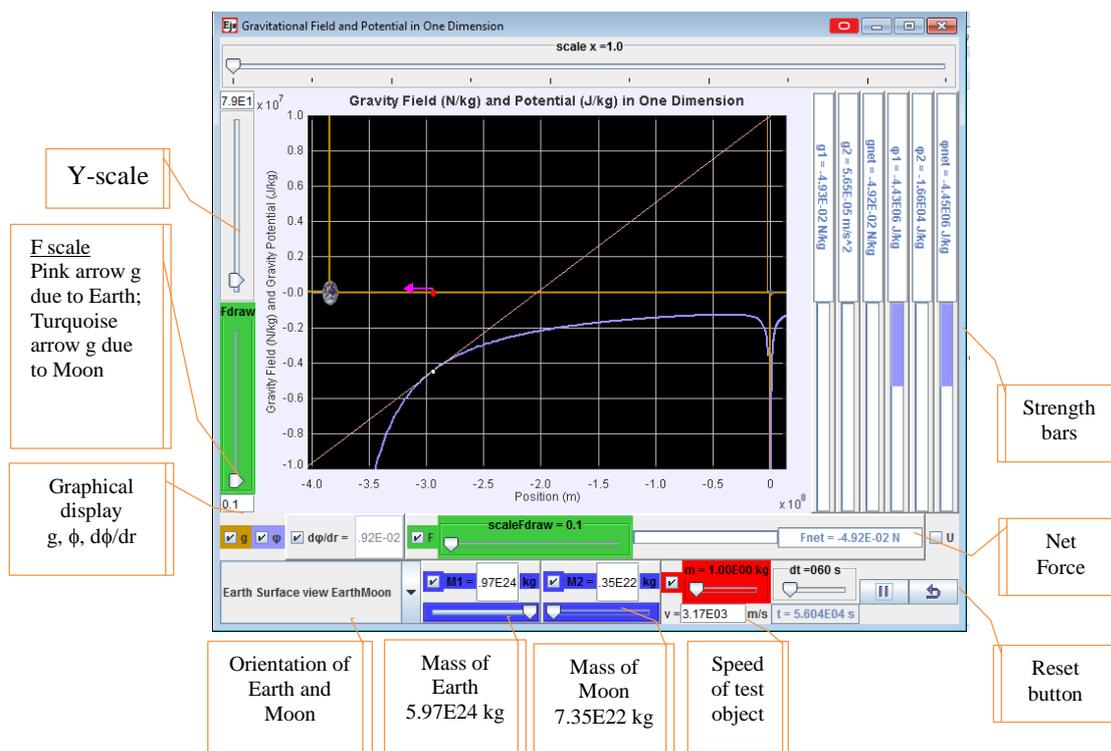
$$v \geq 1.1093 \times 10^4 \text{ m s}^{-1}$$

$$\text{Minimum required speed} = 1.1093 \times 10^4 \text{ m s}^{-1}$$

- (b) Carry out the following ICT inquiry exercise to check whether your answers in **Q7** and **Q15 (a)** can help the body to reach infinity and the surface of Moon respectively.

**Apparatus required:**

- Computer installed with Java runtime and the EJS java applet, titled “**Net gravitational field strength & potential by Earth & Moon Model**”, which can be downloaded from LMS or url: [https://dl.dropbox.com/u/44365627/lookangEJSworkspace/export/ejs\\_GFieldandPotential1Dv7EarthMoon.jar](https://dl.dropbox.com/u/44365627/lookangEJSworkspace/export/ejs_GFieldandPotential1Dv7EarthMoon.jar)



Carry out the following steps to check your answers in **Q7** and **Q15 (a)**.

- Open the EJS java applet titled “**Net gravitational field strength and potential by Earth & Moon Model**”. This model allows you to visualise and investigate the *net gravitational field strength* and *potential* experienced by a test mass ( $m$ ) under the influence of the gravitational fields of Moon ( $M_1$ ) and Earth ( $M_2$ ).
- At the bottom left hand corner, select the “Earth Surface view Earth Moon” so that the Earth is on the left and the Moon on the right, and the test mass is placed on the Earth surface.

To check **Q7**: Required speed from Earth to infinity = \_\_\_\_\_  $\text{m s}^{-1}$

- Uncheck M2 button to hide Moon. Ensure that M1 and m buttons are checked to display the Earth and test mass. Do not adjust the values for the masses as the mass of  $M_1$  is the actual mass of Earth ( $5.97 \times 10^{24}$  kg) and the test mass (red dot) should be kept at 1.00 kg.
- Key in the required speed value to  m/s and press enter.
- Click  to launch the test mass with the required speed and observe whether the test mass ever get pulled back to Earth.

Describe what happens to the arrow’s length and the speed of the object after it has been launched.

The arrow’s length decreases showing that the  $g$  field strength is reducing while the object slows down.

To check **Q15 (a)**: Required speed from Earth to Moon = \_\_\_\_\_  $\text{m s}^{-1}$

- Check M2 button to display Moon. Ensure that M1 and m buttons are still checked. Do not adjust the values for the mass as the mass of  $M_2$  is the actual mass of Moon ( $7.35 \times 10^{22}$  kg).
- Key in the required speed value to  m/s and press enter.
- Click  to launch the test mass with the required speed and observe whether the test mass manages to reach Moon.
- Try launching again with a speed lower than your required speed value to check whether your required speed is the minimum required speed.
- If your required speed value is not the minimum, check your calculation again.

--- End of tutorial ---